EE 508
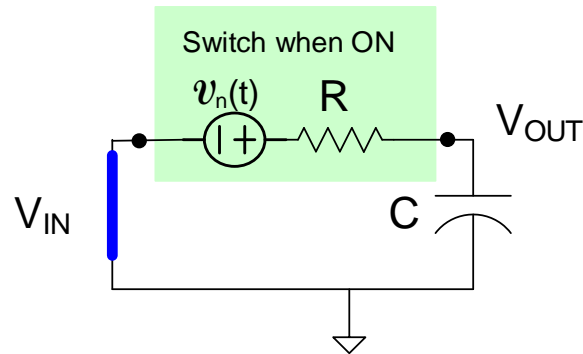# Lecture 27

Integrator Design

- Some other integrator structures
- Metrics for comparing integrators

Noise during sampling phase

Switch when ON

$v_n(t)$ R

V_{OUT}

V_{IN}

C

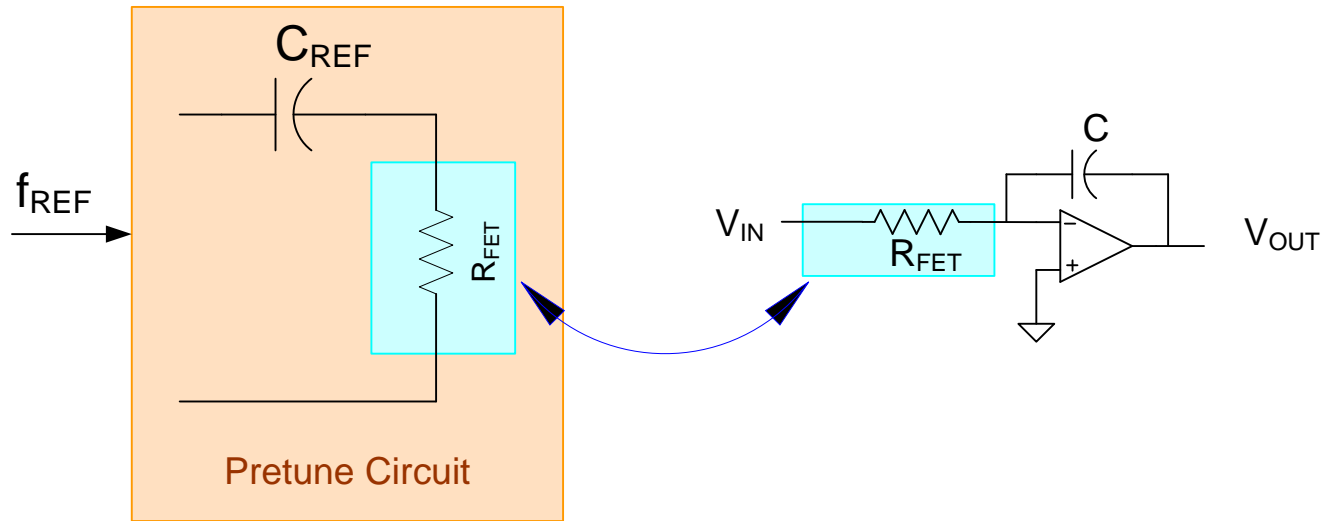$$v_{n_{RMS}} = \sqrt{\frac{kT}{C}}$$

RMS noise voltage on C is independent of the state of the switch

So sampled RMS noise voltage should be same as instantaneous RMS voltage

Highly temperature dependent
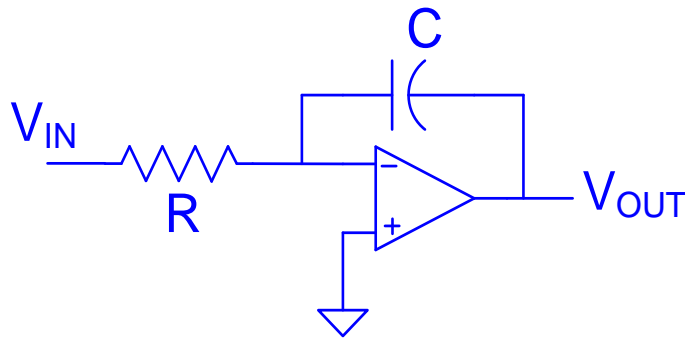
Switched-Resistor Voltage Mode Integrators

Switched-resistor integrator

- Accurate CR_{FET} products is possible
- Area reduced compared to Active RC structure because R_{FET} small
- Single pretune circuit can be used to "calibrate" large number of resistors
- Clock frequency not fast and not critical (but accuracy of f_{REF} is important)
- Since resistors are memoryless elements, no transients associated with switching
- Since filter is a feedback structure, speed limited by BW of op amp

Variants of basic inverting integrator have been considered

Basic Miller Integrator

- Active RC
- MOSFET-C
- OTA-C
- g_m-C
- Switched-Capacitor
- Switched-Resistor

Performance of all is limited by GB of Operational Amplifiers
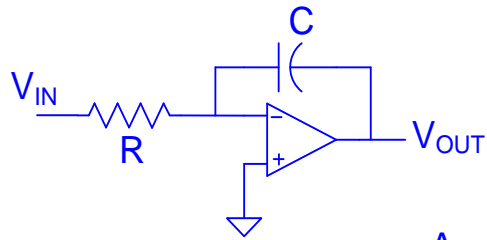
How can integrator performance be improved?

- Better op amps
- Better Integrator Architectures

How can the performance of integrator structures be compared?

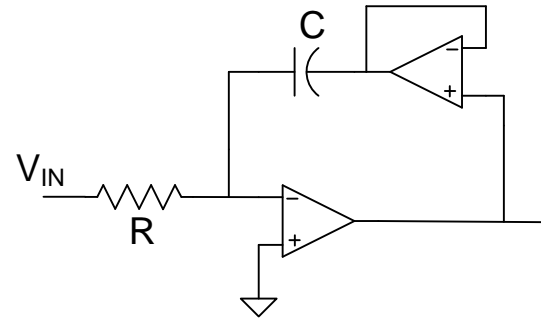Need metric for comparing integrator performance

Are there other integrators in the basic classes that have been considered?
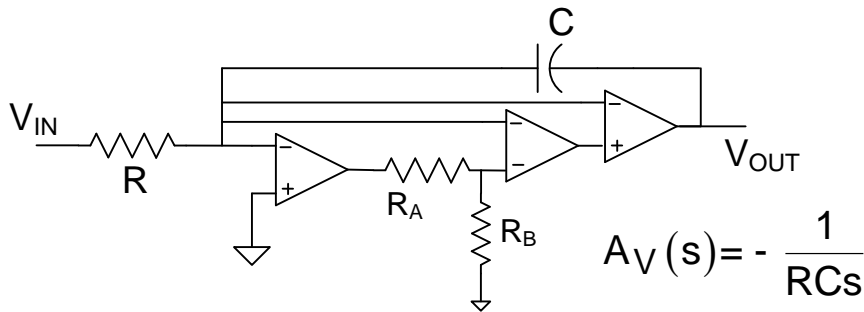

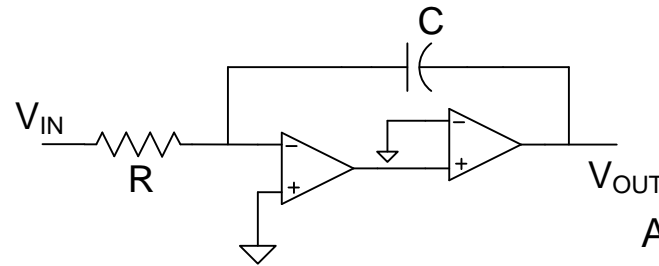$$A_V(s) = -\frac{1}{RCs}$$

Miller Inverting

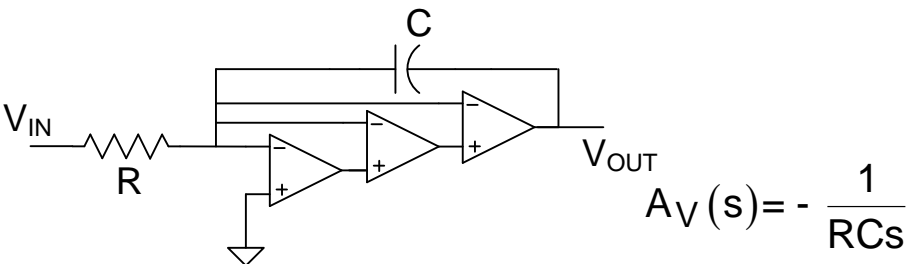$$A_V(s) = -\frac{1}{RCs}$$

High-Q Inverting

$$A_V(s) = -\frac{1}{RCs}$$
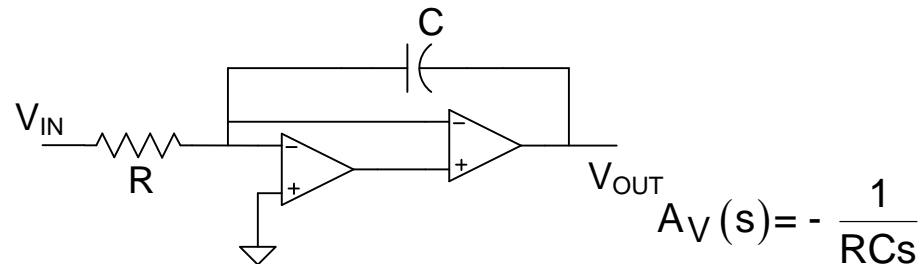
Zero Second Derivative Inverting

$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting

$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting

$$A_V(s) = -\frac{1}{RCs}$$

Zero Sensitivity Inverting

Are there other integrators in the basic classes that have been considered?

Miller Noninverting $\quad A_V(s) = \dfrac{1}{RCs}$

Phase Lead Integrator $\quad A_V(s) = \dfrac{1}{RCs}$

$A_V(s) = \dfrac{1}{RCs}$

Modified High-Q Noninverting

$A_V(s) = \dfrac{1}{RCs}$

High-Q NonInverting

Zero Sensitivity Noninverting $\quad A_V(s) = \dfrac{1}{RCs}$

Modified Miller Noninverting $\quad A_V(s) = \dfrac{1}{RCs}$

Are there other integrators in the basic classes that have been considered?

Miller Noninverting $\quad A_V(s) = \dfrac{1}{RCs}$

Balanced Time Constant Noninverting

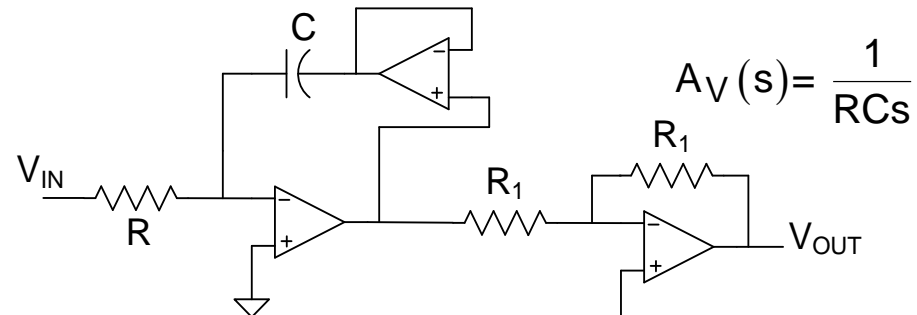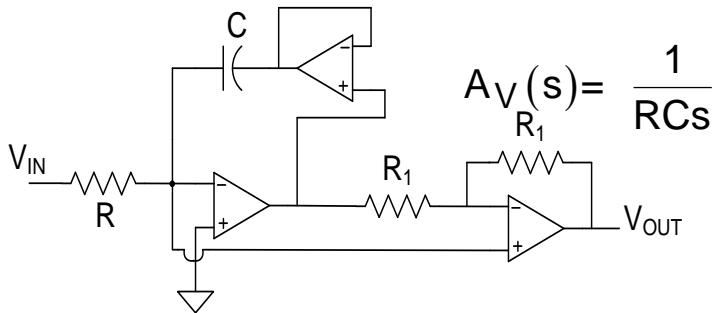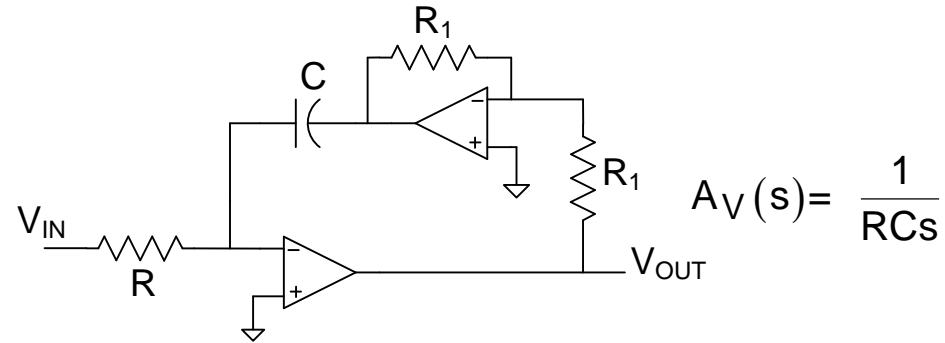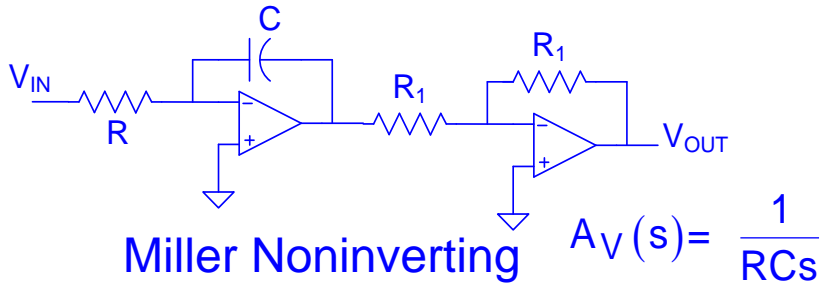$$A_V(s) = \dfrac{1}{RCs}$$

Zero Sensitivity Noninverting
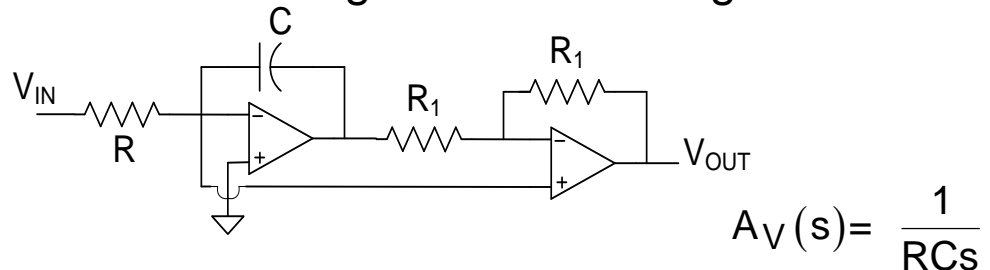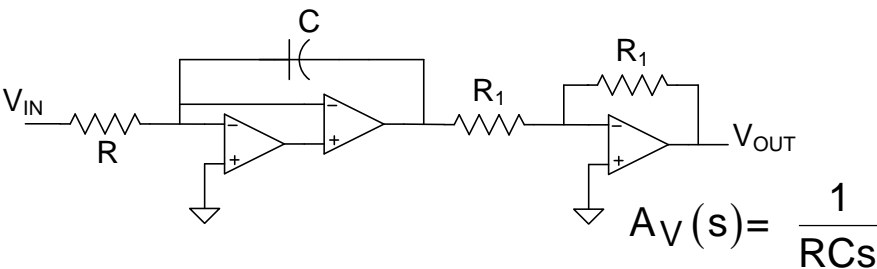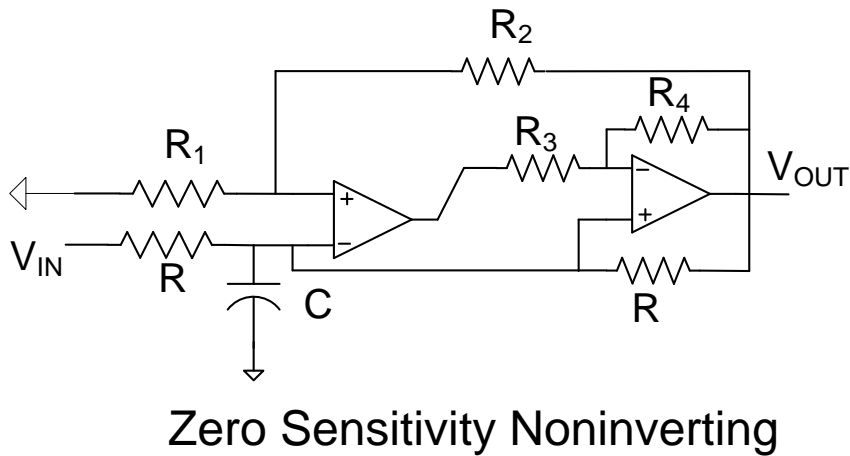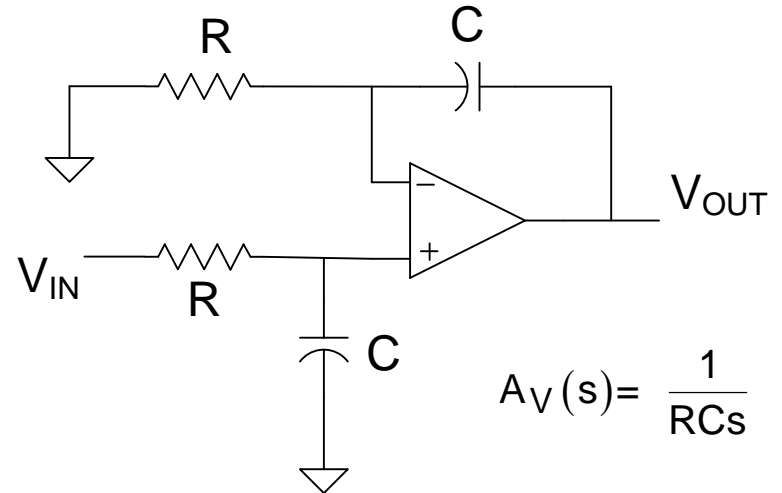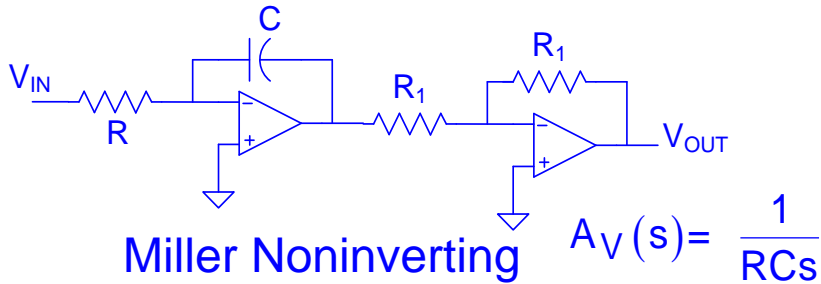
$$A_V(s) = \dfrac{2}{RCs}$$

If $R_1 = R_2$ and $R_3 = R_4$

(note this has a grounded integrating capacitor!)

De Boo Integrator

Consider the Howland Current Source
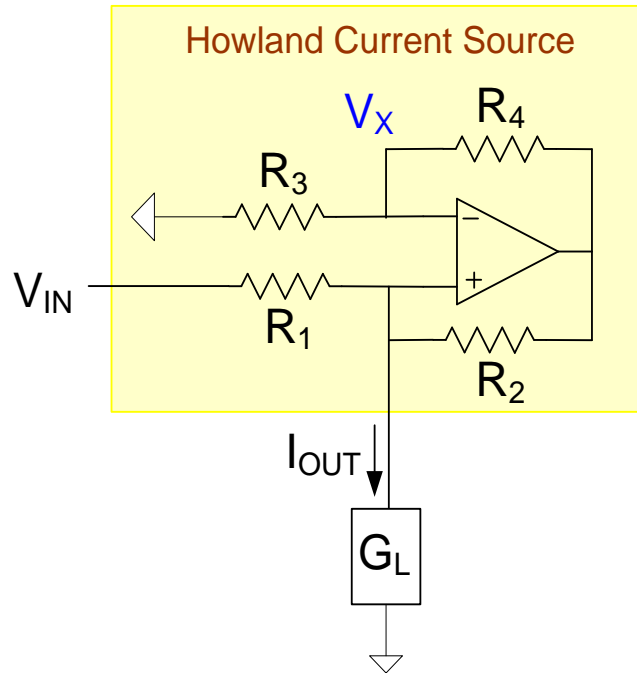
$$I_{OUT} = V_{IN}G_1 + \left[V_X \left(\frac{G_2G_3}{G_4} - G_1 \right) \right]$$

If resistors sized so that $\quad G_1 = \dfrac{G_2G_3}{G_4}$

$$I_{OUT} = V_{IN}G_1$$

If sizing constraints are satisfied, behaves as a constant-current source that can drive a grounded load

DeBoo Integrator

Howland Current Source

V_X

R_2

R_1

V_{IN}

R_1

R_2

I_{OUT}

G_L

$I_{OUT} = V_{IN}G_1$
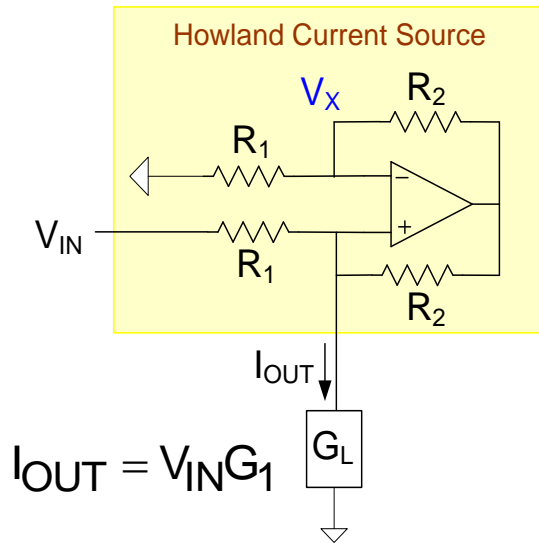
Observe that if a current source drives
a grounded capacitor, then the nodal voltage
on the capacitor is given by

$$V_X = I_X \frac{1}{sC}$$

V_X

I_X

C
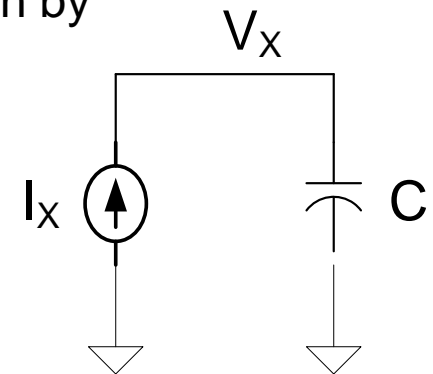
Thus, if I_X is proportional to V_{IN}, the voltage on the capacitor would
be a weighted integral of V_{IN}

De Boo Integrator

V_X

R_2

R_1

R_1

R_2

V_{IN}

I_{OUT}

G_L

$I_{OUT} = V_{IN}G_1$

Howland Current Source

V_X

R_{22}

R_{11}

V_{OUT}

V_{IN}

R_1

R_2

I_{OUT}

C
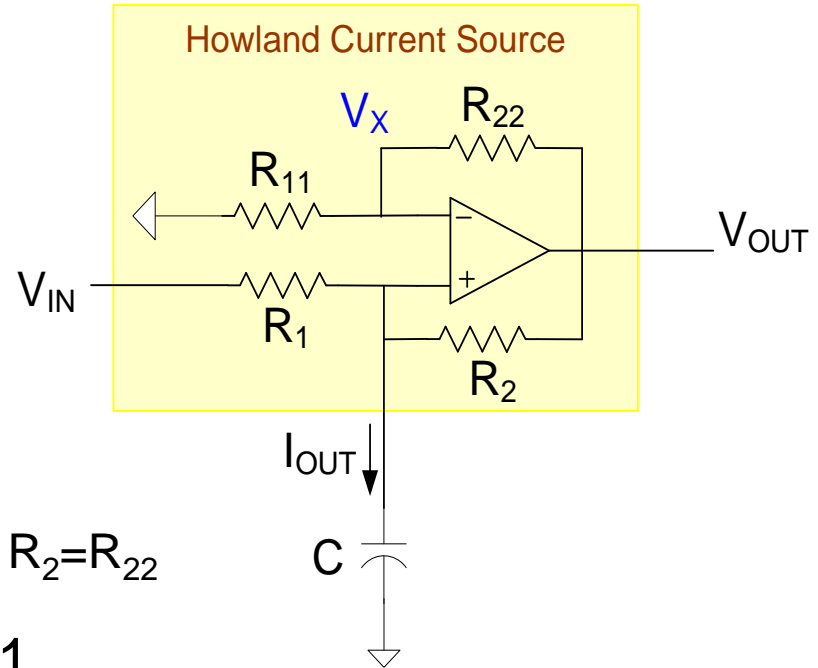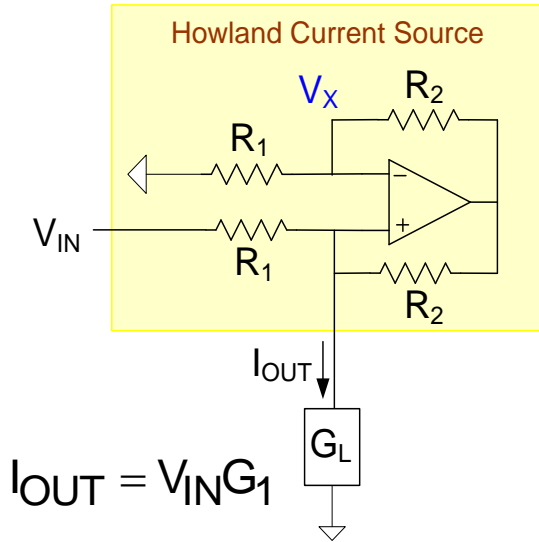
If $R_1 = R_{11}$ and $R_2 = R_{22}$

$$V_X = \frac{V_{IN}}{R_1} \frac{1}{sC}$$

$$V_{OUT} = V_X \left(1 + \frac{R_{22}}{R_{11}}\right) = \frac{V_{IN}}{R_1} \frac{1}{sC} \left(1 + \frac{R_{22}}{R_{11}}\right)$$

$$A_V(s) = \frac{1}{s} \left[\frac{1}{R_1 C} \left(1 + \frac{R_{22}}{R_{11}}\right)\right]$$

De Boo Integrator

Howland Current Source

V_X

R_2

R_1

V_{IN}

R_1

R_2

I_{OUT}

G_L

$I_{OUT} = V_{IN}G_1$

Howland Current Source

V_X

R_2

R_1

V_{IN}

R_1

R_2

V_{OUT}

I_{OUT}

C

De Boo Integrator

$$A_V(s) = \frac{1}{R_1 Cs}\left(1 + \frac{R_2}{R_1}\right)$$

$$I_O(s) = \frac{1}{R_1 C}\left(1 + \frac{R_2}{R_1}\right)$$

Howland Current Source

V_X

R_2

R_1

V_{IN}

R_1

R_2

R_A

R_B

V_{OUT}

I_{OUT}

C
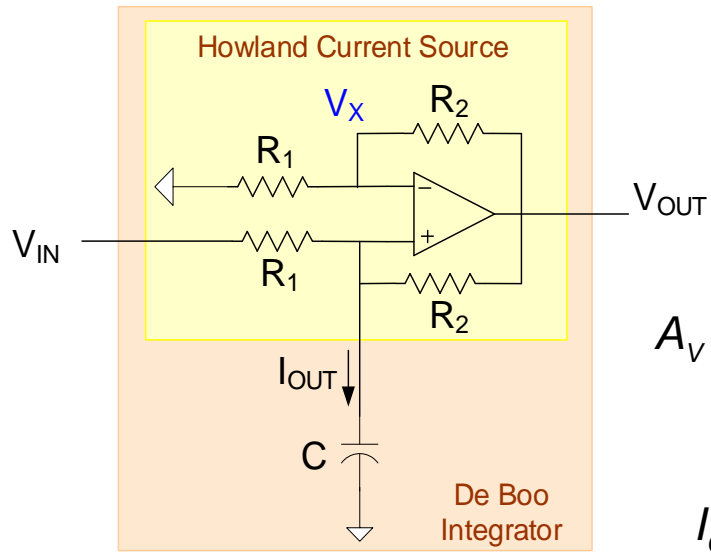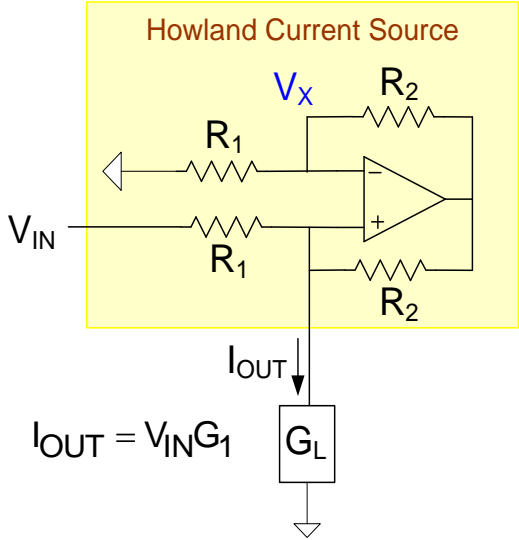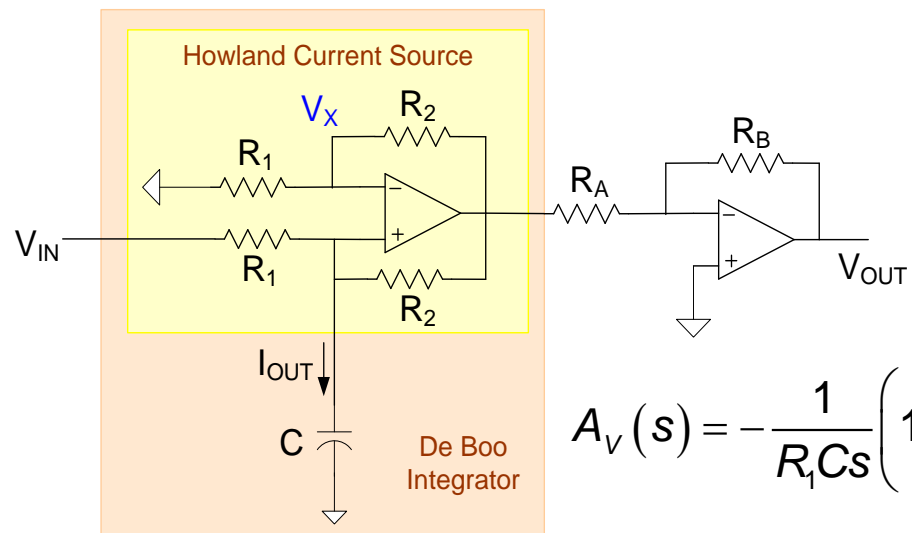
De Boo Integrator

$$A_V(s) = -\frac{1}{R_1 Cs}\left(1 + \frac{R_2}{R_1}\right)\frac{R_B}{R_A}$$

De Boo Integrator

Consider the Sinking Howland Current Source

Howland Current Sink

V_X R_2

R_1

V_{IN}

V_{OUT}

R_3

R_4

I_{OUT}

Z_L

$$I_{OUT}=(V_{OUT}-V_X)G_4-G_3V_X$$
$$V_X(G_1+G_2)=G_1V_{IN}+G_2V_{OUT}$$

$$I_{OUT}=-\frac{G_3+G_4}{G_1+G_2}G_1V_{IN}+\left[\frac{G_1G_4-G_2G_3}{G_1+G_2}\right]V_{OUT}$$

If resistors sized so that $G_1=\dfrac{G_2G_3}{G_4}$

$$I_{OUT}=-G_1V_{IN}$$

If sizing constraints are satisfied, behaves as a sinking constant-current source that can drive a grounded load

De Boo Integrator

Howland Current Sink

V_X
R_2
R_1
V_{IN}
R_3
R_2
V_{OUT}
I_{OUT}
Z_L

Howland Current Source

V_X
R_2
R_1
V_{IN}
R_1
R_2
V_{OUT}
I_{OUT}
C

Inverting De Boo Integrator
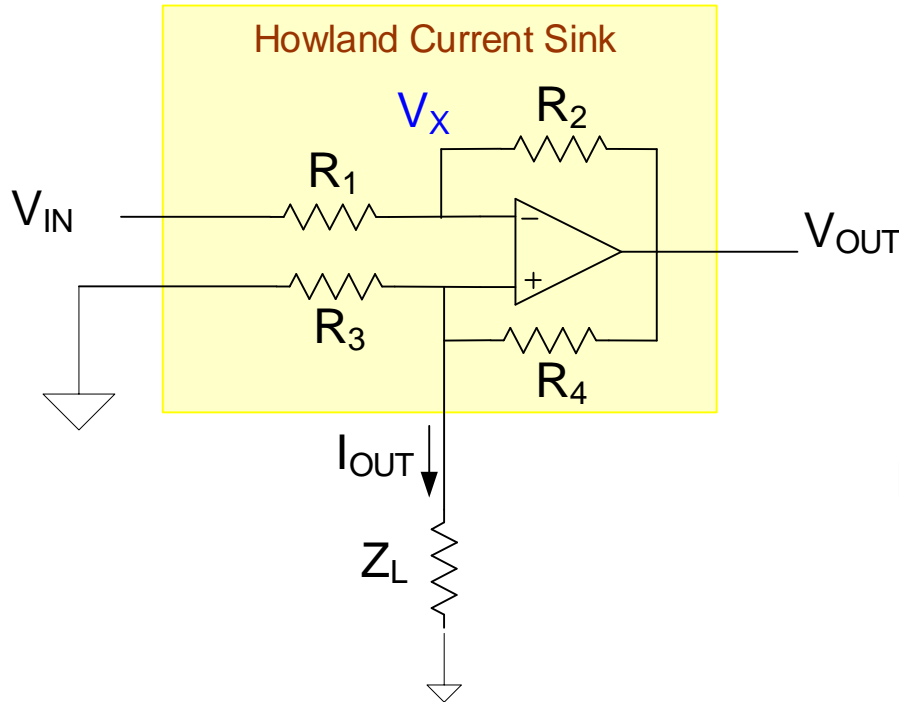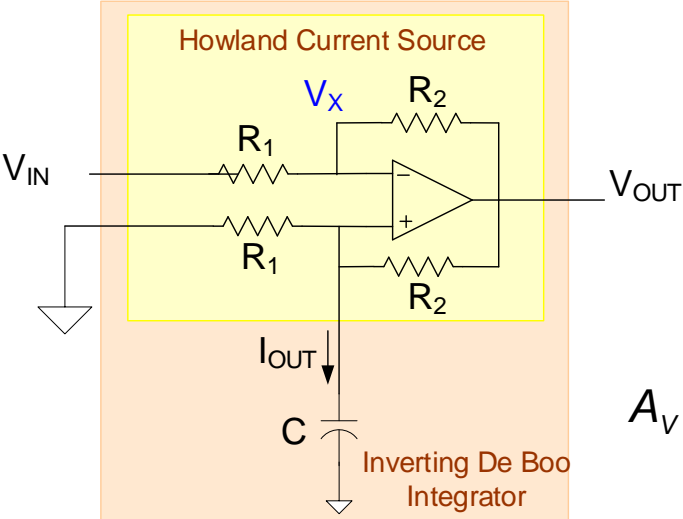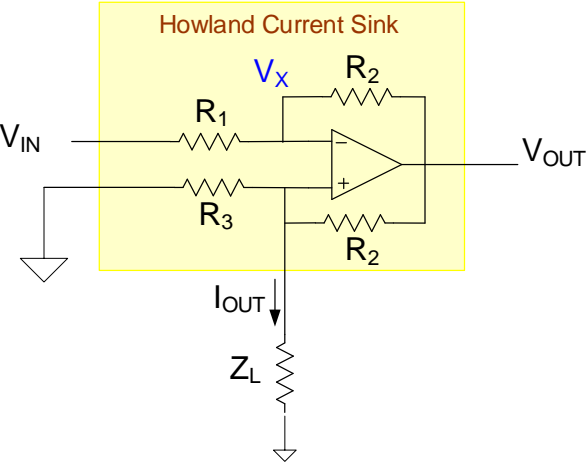
$$A_V(s) = -\frac{1}{R_1 C s}\left(1 + \frac{R_2}{R_1}\right)$$

$$I_O(s) = -\frac{1}{R_1 C}\left(1 + \frac{R_2}{R_1}\right)$$

$$I_{OUT} = -G_1 V_{IN}$$

Observations:

Many different integrator architectures ideally provide the same gain

Similar observations can be made for other classes of integrators

Are there benefits or limitations for using the different integrator structures?
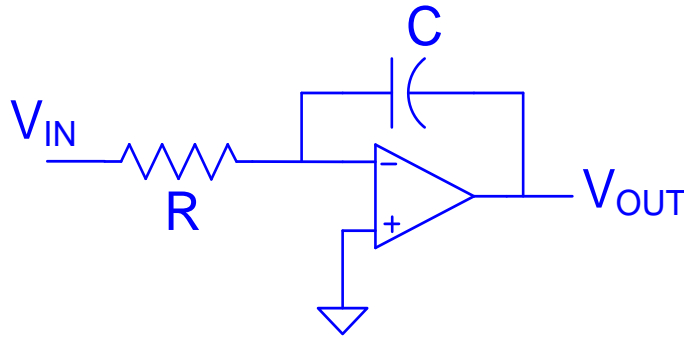
How can the performance of an integrator be characterized and how can integrators be compared?

Before considering performance of the integrators consider current-mode integrators

Basic Miller Integrators

$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{RCs}$$

Voltage Mode
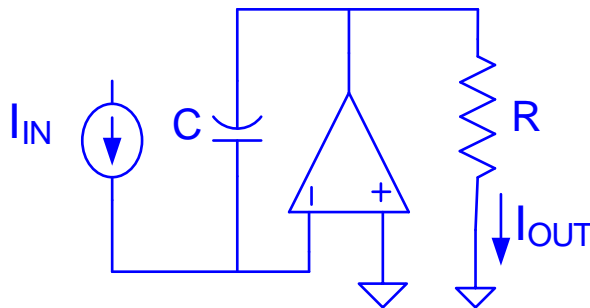
$$\frac{I_{OUT}}{I_{IN}} = -\frac{1}{RCs}$$

Current Mode

- Both use the same number of components and same type of components
- Many authors claim the current-mode integrator offers several major benefits to the voltage-mode integrator
- Will discuss the differences in more detail later

Basic Miller Integrators
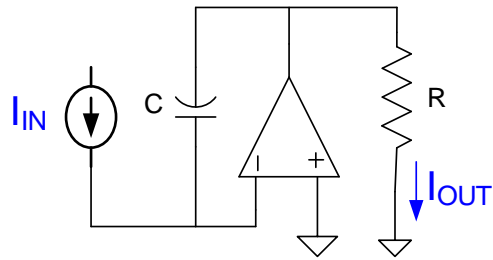
$$\frac{I_{OUT}}{I_{IN}} = -\frac{1}{RCs}$$
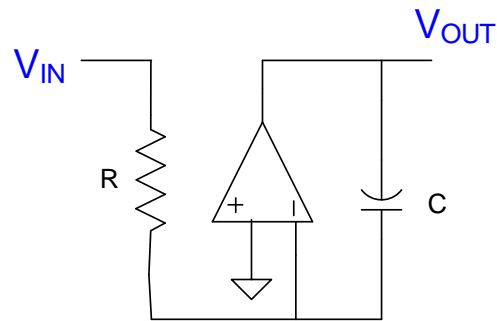
$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{RCs}$$
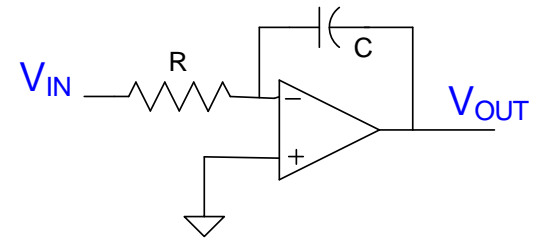
$$\frac{V_{OUT}}{V_{IN}} = -\frac{1}{RCs}$$

How does the one in the middle differ from the one on the right?

Preliminary Comparison of Current Mode Integrators with Voltage Mode Integrators

Consider Integrator Cascade

Preliminary Comparison of Current Mode Integrators with Voltage Mode Integrators

Consider Integrator Cascade
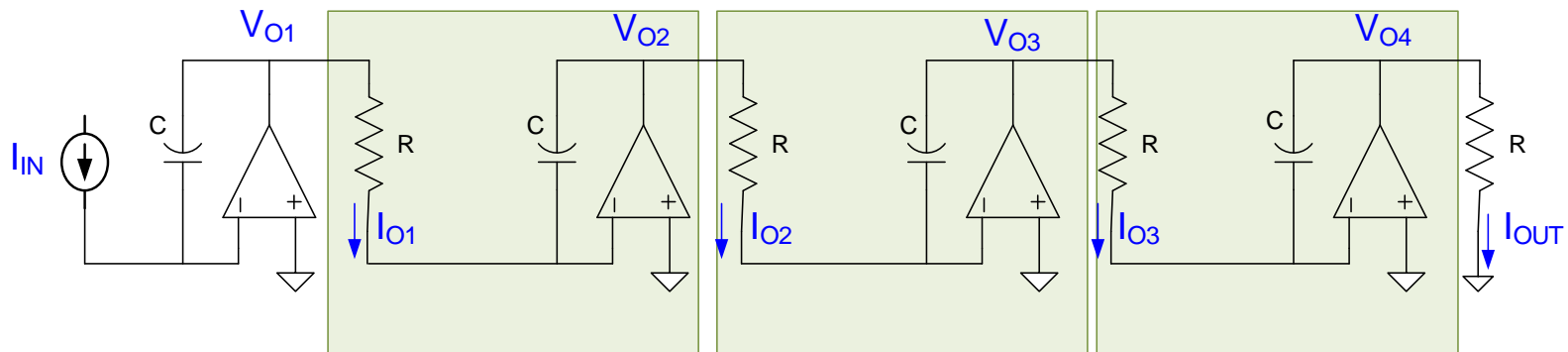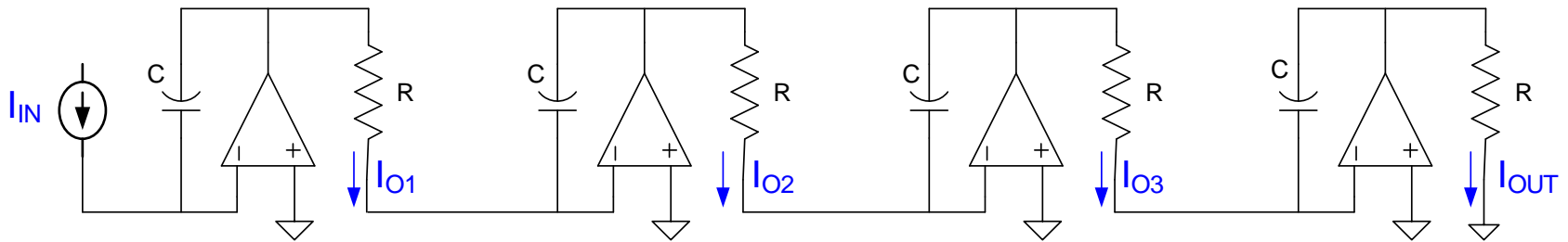
If connected in a loop, structures are identical !

How can the performance of an integrator be characterized and how can integrators be compared?

Consider Ideal Integrator Gain Function

$$A_V(s) = \frac{I_0}{s} \qquad A_V(j\omega) = \frac{I_0}{j\omega}$$

Ideal Integrator
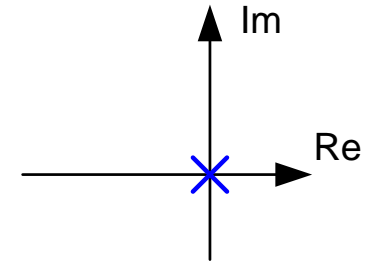
Consider a nonideal integrator Gain Function

$$A_V(s) = \frac{\alpha I_{01}}{s+\alpha} A_{OO}(s)$$

Nonideal Integrator
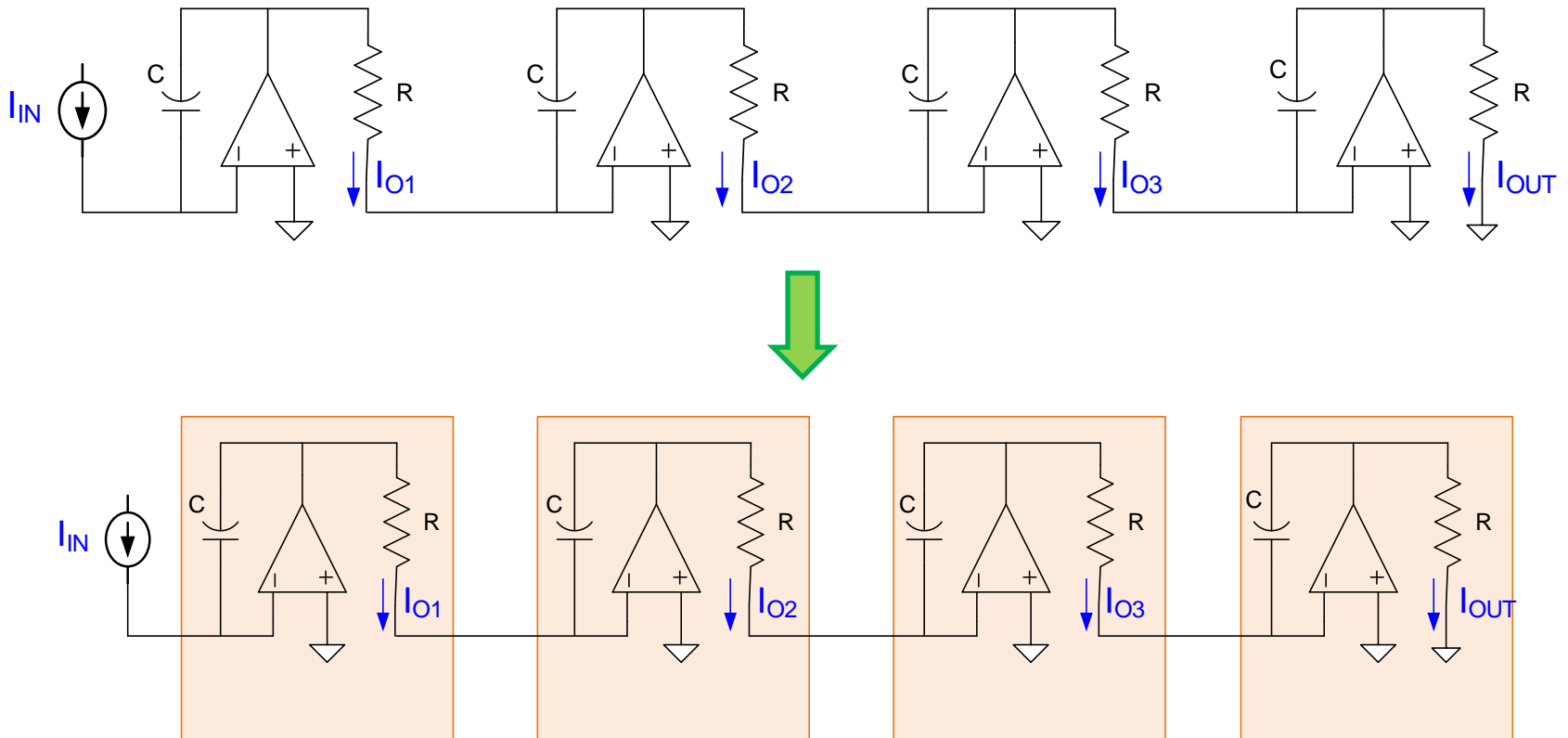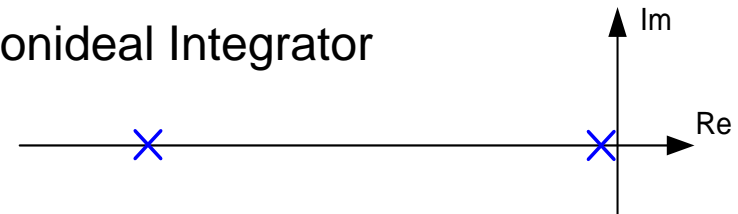

Key characteristics of an ideal integrator:

- Magnitude of the gain at $I_0 = 1$
- Phase of integrator always 90°
- Gain decreases with $1/\omega$

Are any of these properties more critical than others?

In many applications:

Key property of ideal integrator is a phase shift of 90° at frequencies around I_0!

How can the performance of an integrator be characterized and how can integrators be compared?

Is stability of an integrator of concern?

Ideal Integrator

Im

Re
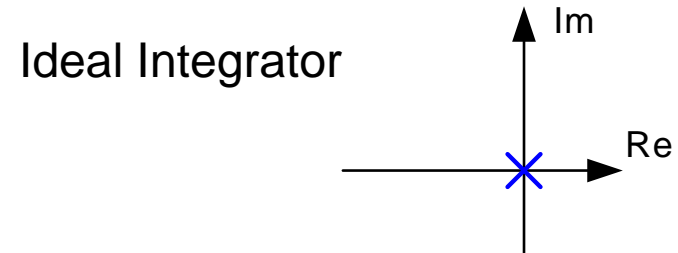
- Ideal integrator is not stable
- Integrator function is inherently ill-conditioned
- Integrator is almost never used open-loop
- Stability of integrator not of concern, stability of filter using integrator is of concern
- Some integrators may cause unstable filters, others may result in stable filters
- Instability in filter because desired poles move in RHP is of little concern since the filter performance would be unacceptable long before the stability became an issue
- Instability in filter due to parasitic poles is of concern but not a problem in most circuits

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$A_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

where $R(\omega)$ and $X(\omega)$ are real and represent the real and imaginary parts of the denominator respectively

$$\text{Phase} = -\tan^{-1}\left|\left(\frac{X(\omega)}{R(\omega)}\right)\right|$$

Ideally $R(\omega) = 0$

Definition: The Integrator Q factor is the ratio of the imaginary part of the denominator to the real part of the denominator

$$Q_{INT} = \left|\left(\frac{X(\omega)}{R(\omega)}\right)\right|$$

Typically most interested in Q_{INT} at the nominal unity gain frequency of the integrator

How can the performance of an integrator be characterized and how can integrators be compared?

Express $A_V(j\omega)$ as

$$I_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)}$$

Lead/Lag Characteristics for <u>Inverting</u> Integrators (inverting gain at ω_0 should be 1 at angle of 90°)

Phase Lag Integrator — $I(j\omega_0)$

Ideal Phase Integrator — $I(j\omega_0)$

$I(j\omega_0)$ — Phase Lead Integrator
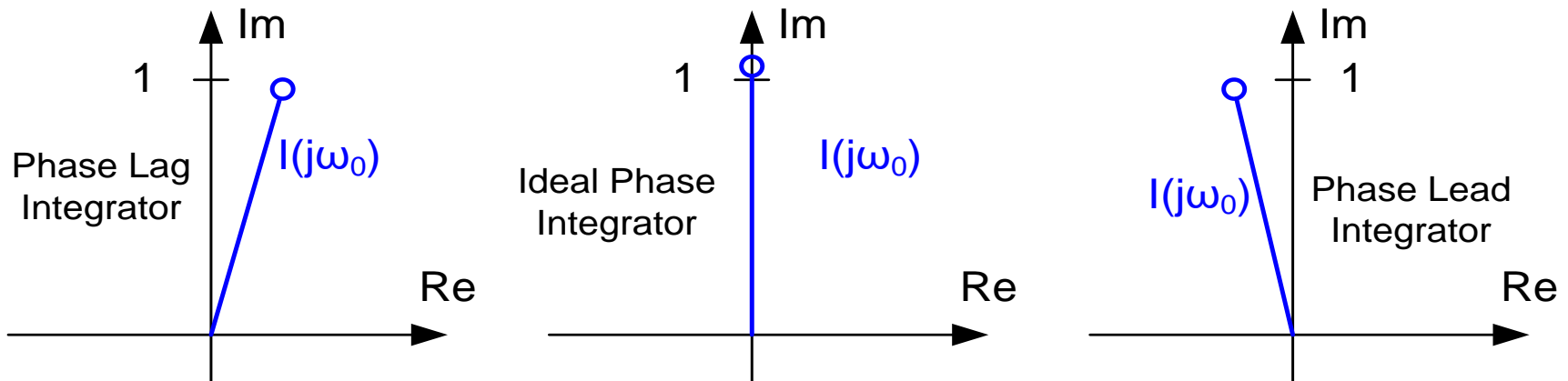
ω_0 is the unity gain frequency of the integrator

For Phase Lag Integrators, $R(\omega)$ is negative
For Phase Lead integrators, $R(\omega)$ is positive

How can the performance of an integrator be characterized and how can integrators be compared?
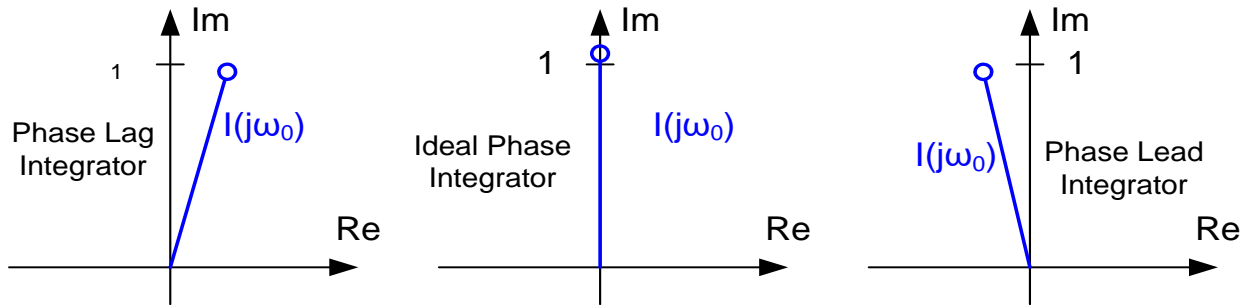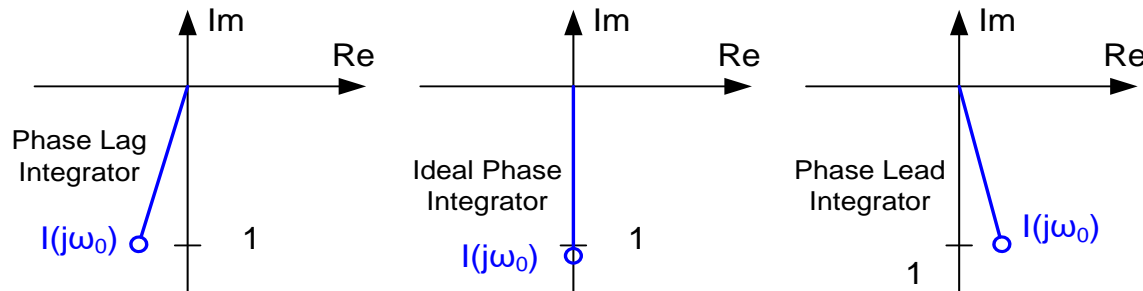
Lead/Lag Characteristics for Inverting Integrators

$$I_V(j\omega) = \frac{-1}{R(\omega) + jX(\omega)}$$

For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs.   For Phase Lead integrators, $R(\omega)$ and $X(\omega)$ have the same sign.   Phase shift ideally 90°

Lead/Lag Characteristics for Noninverting Integrators (noninverting gain at ω_0 should be 1 at angle of -90°)
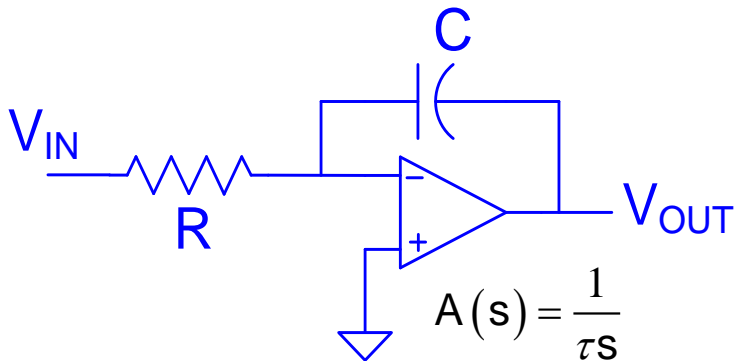

$$I_V(j\omega) = \frac{1}{R(\omega) + jX(\omega)}$$

For Phase Lag Integrators, $R(\omega)$ and $X(\omega)$ have opposite signs.   For Phase Lead integrators, $R(\omega)$ and $X(\omega)$ have the same sign.   Phase shift ideally 270°

Integrator Q Factor

Consider Miller Inverting Integrator

$$A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)}$$

$$A_V(s) = \frac{-1}{RCj\omega + \tau j\omega(1 + RCj\omega)}$$

$$A_V(s) = \frac{-1}{-\tau\omega^2 RC + j(\omega[RC + \tau])}$$

Normalizing by $\omega_n = \omega RC$ and $\tau_n = \tau/RC = I_{0n}/GB$

$$A_V(s) = \frac{-1}{-\tau_n\omega_n^2 + j(\omega_n[1 + \tau_n])}$$
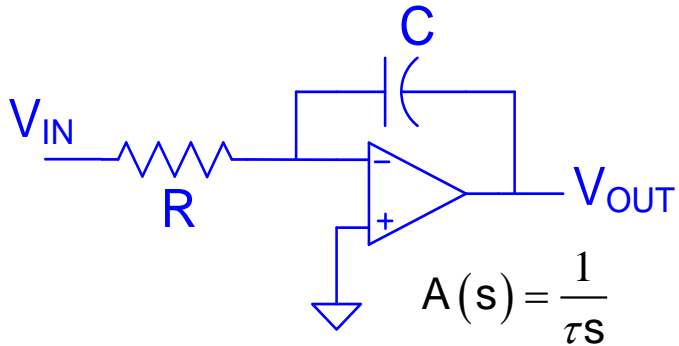
Observe this integrator has excess phase shift (more than 90° in the denominator) at all frequencies

Integrator Q Factor

$$A_V(j\omega) = \frac{\pm 1}{R(\omega) + jX(\omega)} \qquad\qquad Q_{INT} = \left(\frac{X(\omega)}{R(\omega)}\right)$$

Consider Miller Inverting Integrator

$$A_V(s) = \frac{-1}{-\tau_n\omega_n^2 + j(\omega_n[1+\tau_n])}$$

$$Q_{INT} = -\frac{\omega_n[1+\tau_n]}{\tau_n\omega_n^2} \cong -\frac{1}{\tau_n\omega_n}$$

$$A(s) = \frac{1}{\tau s} \qquad Q_{INT}\big|_{\omega_N=1} \cong -\frac{1}{\tau_n\omega_n}\bigg|_{\omega_N=1} \cong -\frac{1}{\tau_n}\bigg| = -\frac{GB}{\omega_0} = -A(I_{0N}) = -A$$

Since the phase is less than 90°, the Miller Inverting Integrator is a Phase Lag Integrator and Q_{INT} is negative
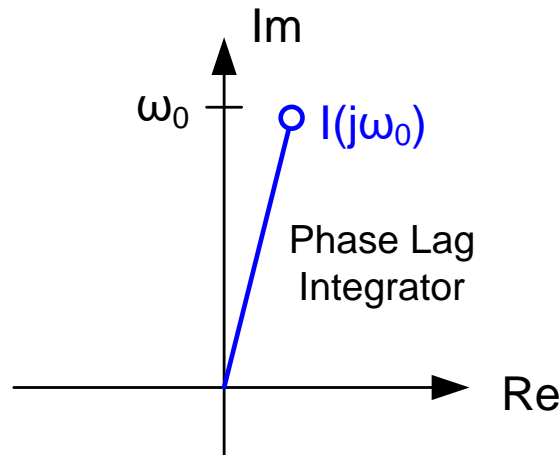
Integrator Pole Locations

Consider Miller Inverting Integrator
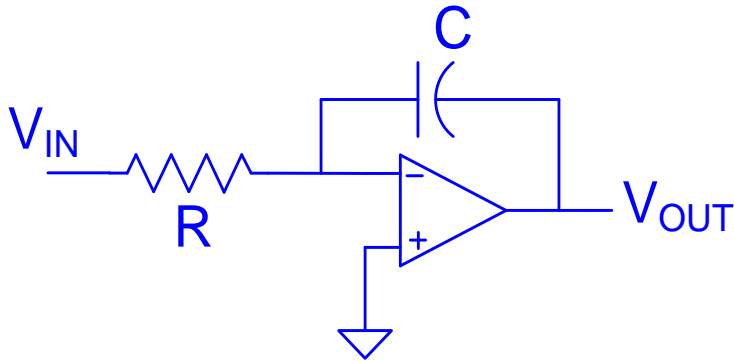
$$A_V(s) = \frac{-1}{RCs + \tau s(1 + RCs)}$$

$$I_0 = 1/(RC)$$

Poles at s=0 and s = $-I_0(1 + GB/I_0) \simeq$ -GB
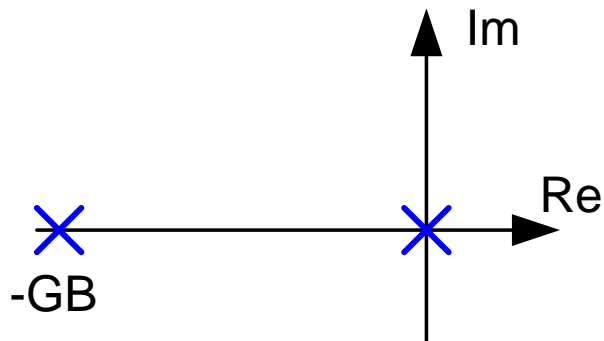
Is the integrator Q factors simply a metric or does it have some other significance?
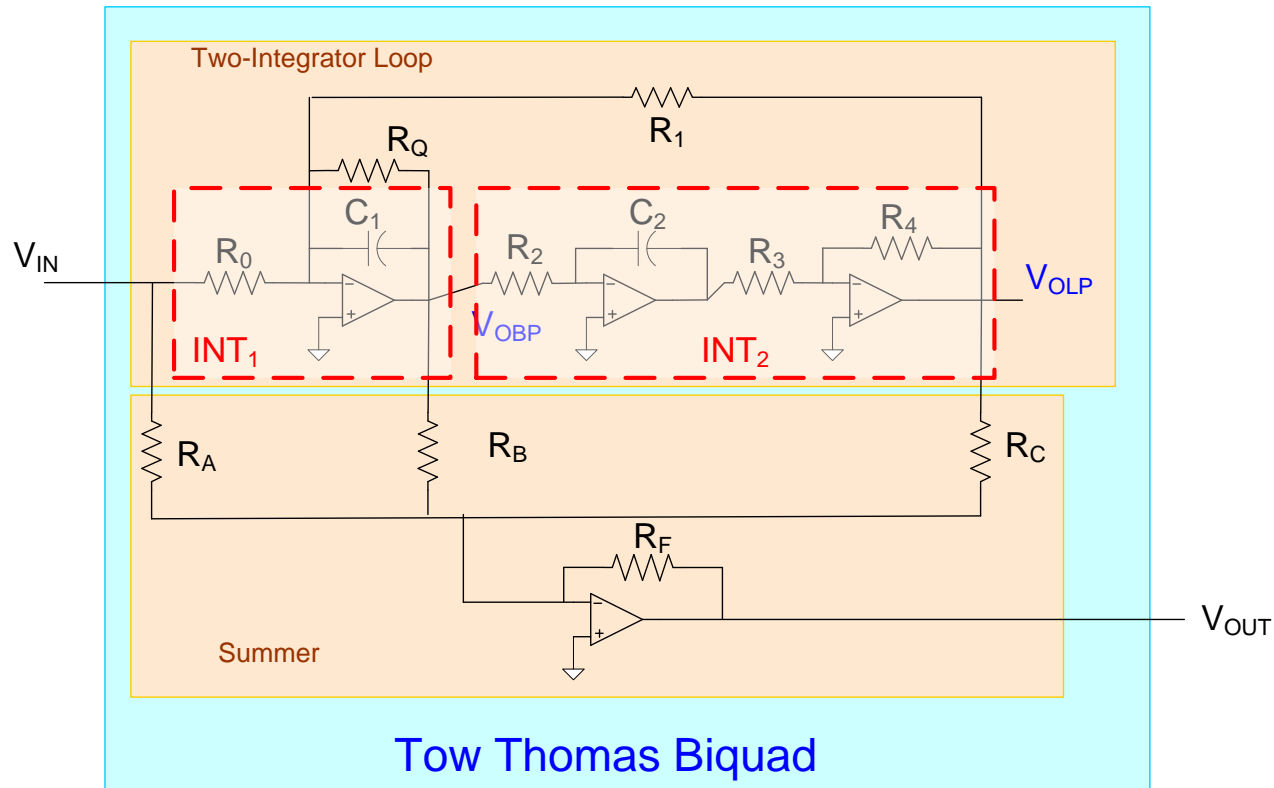
It can be shown that the pole Q for the TT Biquad can be approximated by

$$Q_P \cong \cfrac{1}{\cfrac{1}{Q_{PN}} + \cfrac{1}{Q_{INT1}} + \cfrac{1}{Q_{INT2}}}$$

where Q_{INT1} and Q_{INT2} are evaluated at $\omega = \omega_o$

Is the integrator Q factors simply a metric or does it have some other significance?
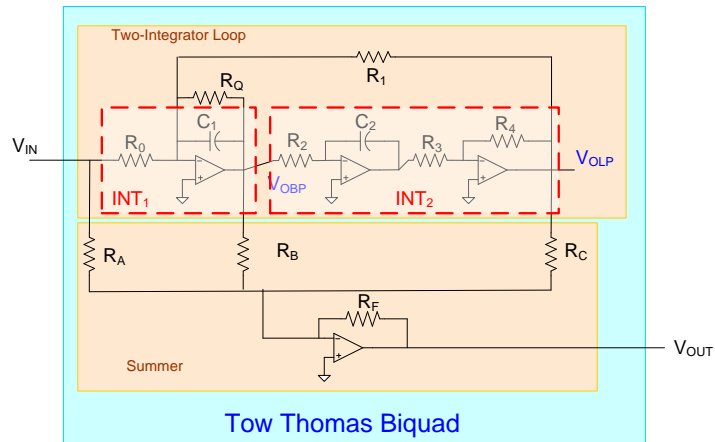

It can be shown that the pole Q for the TT Biquad can be approximated by
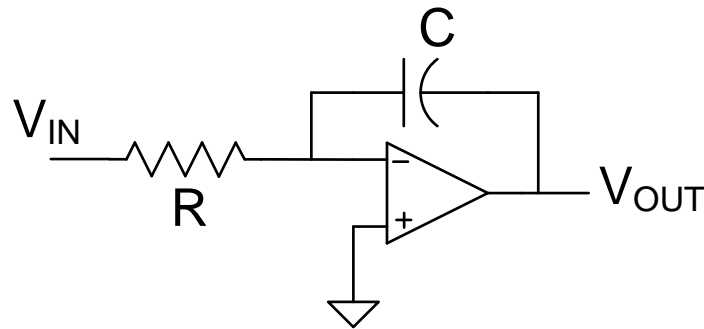
$$Q_P \cong \cfrac{1}{\cfrac{1}{Q_{PN}} + \cfrac{1}{Q_{INT1}(\omega_0)} + \cfrac{1}{Q_{INT2}(\omega_0)}}$$

Similar expressions for other second-order biquads

Observe that the integrator Q factors adversely affect the pole Q of the filter

Observe that if Q_{INT1} and Q_{INT2} are of opposite signs and equal magnitudes, nonideal effects of integrator can cancel

What can be done to correct the phase problems of an integrator?

$$A_V(s) = \frac{-1}{RCs + \tau s(1+RCs)}$$

One thing that can help the Miller Integrator is phase-lead compensation
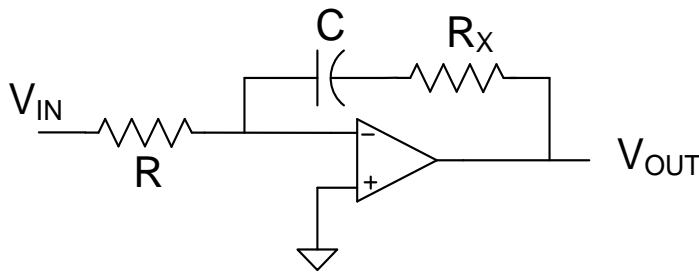
$$A_V(s) = \frac{-(1+R_X Cs)}{RCs + \tau s(1+[R+R_X]Cs)}$$

$$A_V(s) = \frac{-(1+RC_X s)}{RCs + \tau s(1+R[C+C_X]s)}$$

R_x and C_x will add phase-lead by introduction of a zero

R_X and C_X will be small components

Integrator Q Factor

Consider Miller Noninverting Integrator

$$A_V(s) = \frac{-1}{RCs + \tau s(1+RCs)} \cdot \frac{-1}{1+2\tau s}$$

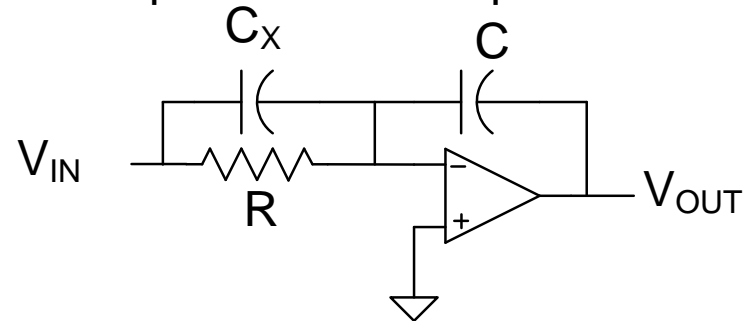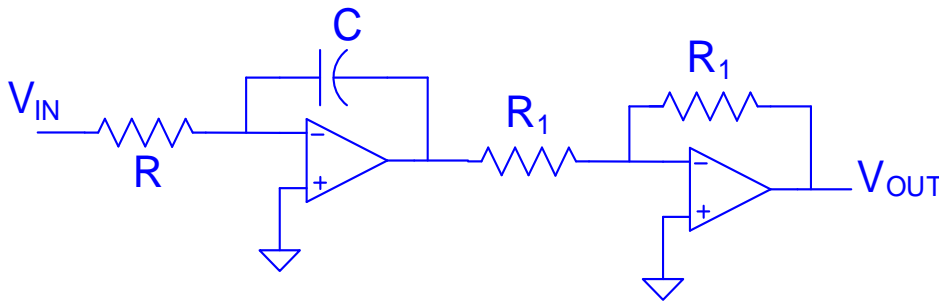$$A_V(j\omega) = \frac{1}{-3\tau RC\omega^2 + j[\omega RC(1+2\tau\omega)]}$$

$$A_V(j\omega) \cong \frac{1}{-3\tau RC\omega^2 + j\omega RC}$$

Observe this integrator has excess phase shift (more than 90º in the denominator) at all frequencies

$$Q_{INT} \cong \frac{\omega RC}{-3\tau RC\omega^2} = \frac{-1}{3\tau\omega}$$

$$Q_{INT} \cong \frac{-1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1}{3}|A(j\omega)|$$

Note: The Miller Noninverting Integrator has a modestly poorer Q_{INT} than the Miller Inverting Integrator

Example:

If $f_0 = 10KHz$, GB=1MHz, $Q_{NOM}=10$, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.
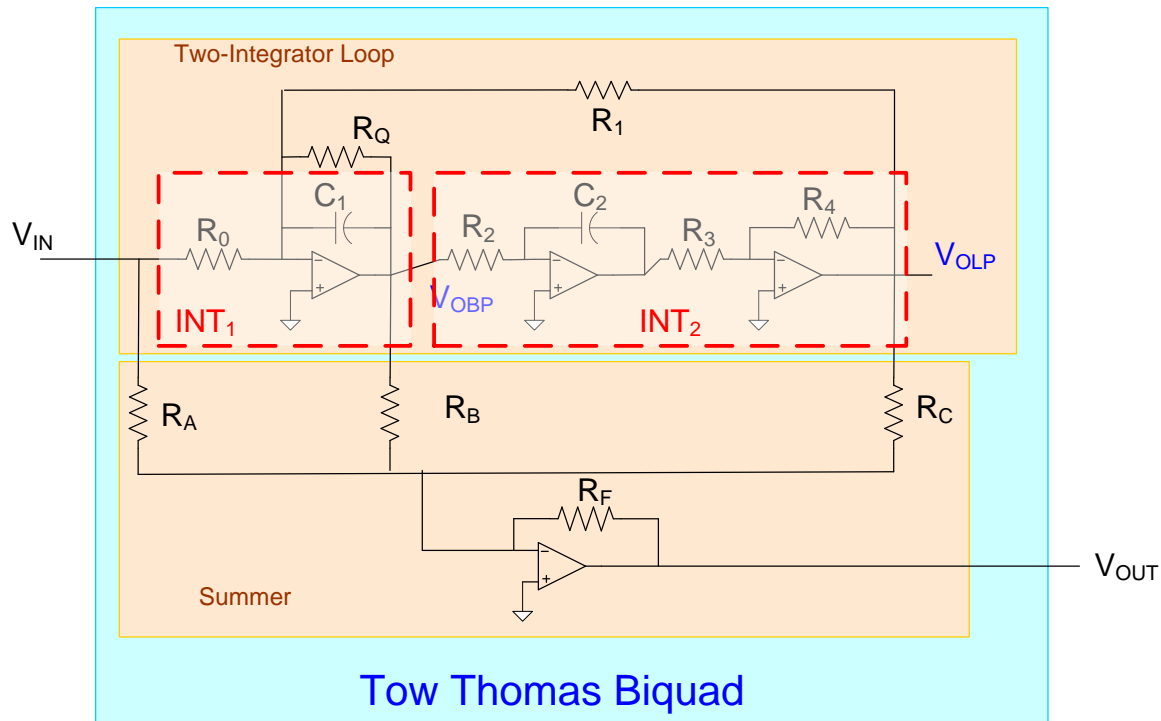
Tow Thomas Biquad

$$Q_P \cong \cfrac{1}{\cfrac{1}{Q_{PN}} + \cfrac{1}{Q_{INT1}} + \cfrac{1}{Q_{INT2}}}$$

Example:

If f_0=10KHz, GB=1MHz, Q_{NOM}=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}} + \frac{1}{Q_{INT1}} + \frac{1}{Q_{INT2}}}$$

$$Q_{INT1} = \text{-A} = -\frac{GB}{\omega} = -\frac{1MHz}{10KHz} = -100$$

$$Q_{INT2} \cong -\frac{1}{3}\left|A(j\omega)\right| = \frac{-1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1MHz}{3 \cdot 10KHz} = \text{-33}$$

$$Q_P \cong \frac{1}{\frac{1}{10} - .01 - .033} = 17.5$$

Note the nonideal integrators cause about a 75% shift in Q_P

Note that 3 times as much of the shift is due to the noninverting integrator as is due to the inverting integrator!

Similar effects of the integrators will be seen on other filter structures

Example:

If f_0=10KHz, GB=1MHz, Q_{NOM}=10, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\frac{1}{Q_{PN}}+\frac{1}{Q_{INT1}}+\frac{1}{Q_{INT2}}}$$

$$Q_{INT1} = \text{-A} = -\frac{GB}{\omega} = -\frac{1MHz}{10KHz} = -100$$

$$Q_{INT2} \simeq -\frac{1}{3}|A(j\omega)| = \frac{-1}{3}\left(\frac{GB}{\omega}\right) = -\frac{1MHz}{3 \cdot 10KHz} = \text{-33}$$

$$Q_P \cong \frac{1}{\frac{1}{10}\text{-.01-.033}} = 17.5$$

How can the problem be solved?

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

Example:

If $f_0 = 10\text{KHz}$, GB=1MHz, $Q_{NOM}=10$, estimate the pole Q for the Tow-Thomas Biquad if the Miller Integrator and the Miller Noninverting Integrators are used. Also determine the relative degradation in performance due to each of the integrators.

$$Q_P \cong \frac{1}{\dfrac{1}{Q_{PN}} + \dfrac{1}{Q_{INT1}} + \dfrac{1}{Q_{INT2}}}$$

$$Q_{INT1} = -100 \qquad Q_{INT2} \simeq -33$$

How can the problem be solved?

Phase Compensation of INT1
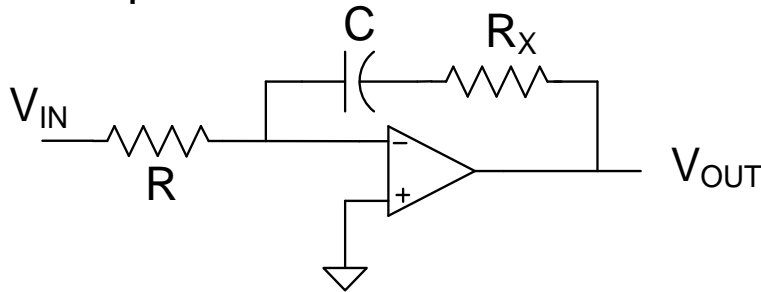
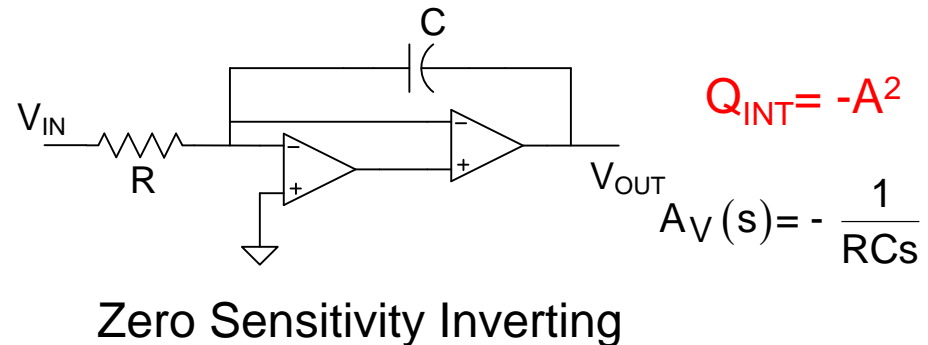
$$A_V(s) = \frac{-(1+R_X Cs)}{RCs + \tau s(1+[R+R_X]Cs)}$$

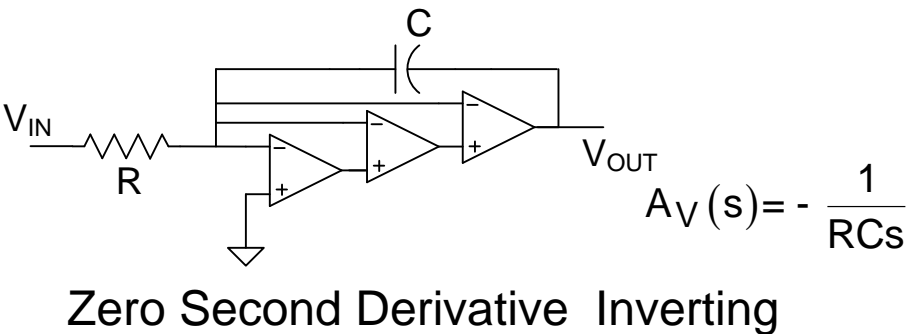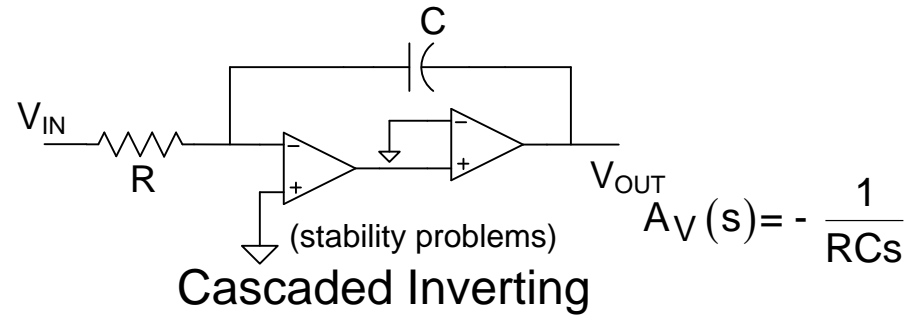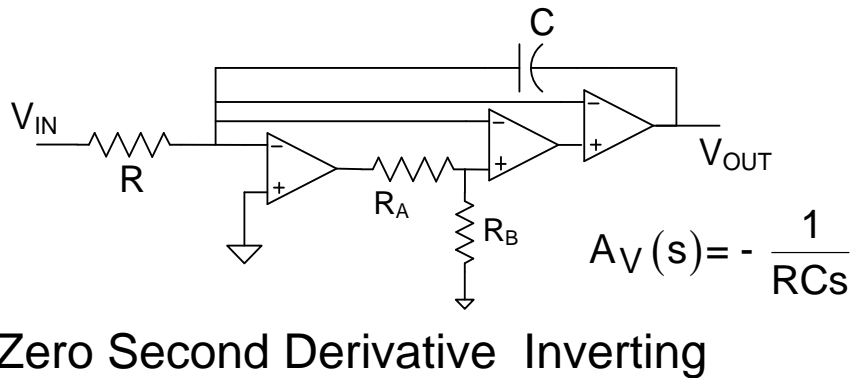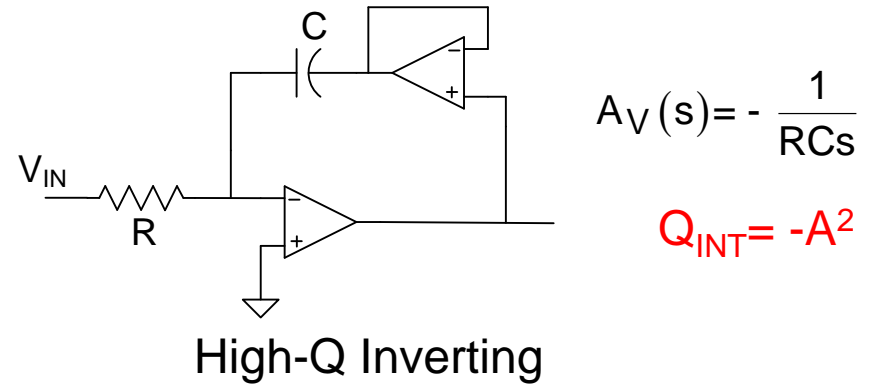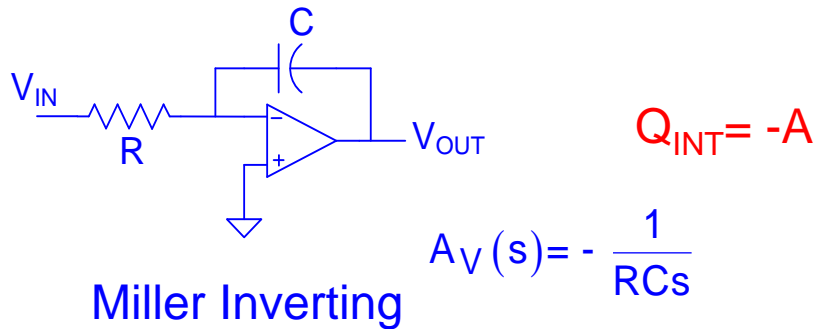$$Q_{INT1} = \frac{-GB/\omega}{1 - GB \bullet CR_X}$$

Pick R_x so that $Q_{INT1}=33$ at $\omega=1/(RC)$

Solving, obtain $CR_x=4/GB$

Useful for hand calibration but not practical for volume production because of variability in components

What are the integrator Q factors for other integrators that have been considered?

Miller Inverting

$$Q_{INT} = -A$$

$$A_V(s) = -\frac{1}{RCs}$$

High-Q Inverting

$$A_V(s) = -\frac{1}{RCs}$$

$$Q_{INT} = -A^2$$

Zero Second Derivative Inverting

$$A_V(s) = -\frac{1}{RCs}$$

Cascaded Inverting

(stability problems)

$$A_V(s) = -\frac{1}{RCs}$$

Zero Second Derivative Inverting

$$A_V(s) = -\frac{1}{RCs}$$

Zero Sensitivity Inverting

$$Q_{INT} = -A^2$$

$$A_V(s) = -\frac{1}{RCs}$$

What are the integrator Q factors for other integrators that have been considered?

V_{IN} ⎓R⎓ —[C]— op-amp → V_{OUT}

Miller Inverting

$$Q_{INT} = -A$$

$$A_V(s) = -\frac{1}{RCs}$$

Howland Current Source

V_X, R_2, R_1, V_{IN}, R_1, R_2, V_{OUT}

I_{OUT} ↓

C_X

De Boo Integrator
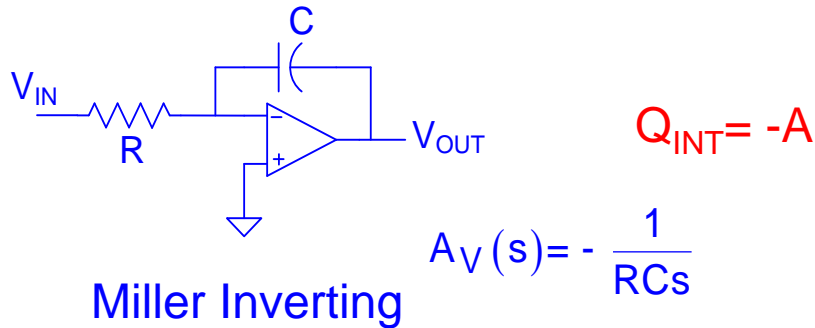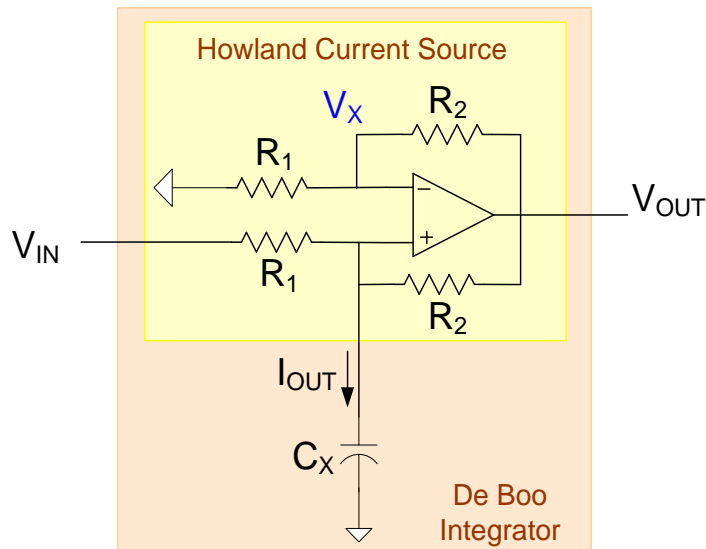
$$A_V(s) = \frac{1 + \dfrac{R_2}{R_1}}{R_1Cs + \tau_1 s\left(1 + \dfrac{R_2}{R_1}\right)\left(1 + \dfrac{R_1}{R_2} + sCR_1\right)}$$

$$Q_{INT} = -\frac{A}{1 + \dfrac{R_2}{R_1}}$$

If $R_1 = R_2 = R$

$$A_V(s) = \frac{2}{RCs}$$

$$Q_{INT} = -\frac{A}{2}$$

What are the integrator Q factors for other integrators that have been considered?

Miller Noninverting

$$Q_{INT} \simeq -\frac{1}{3}\left|A(j\omega)\right|$$
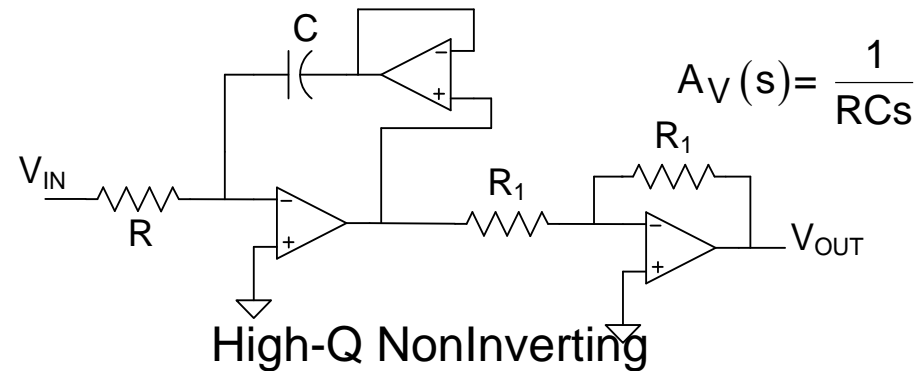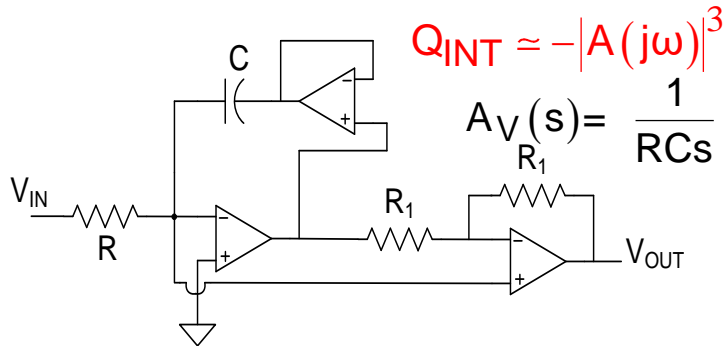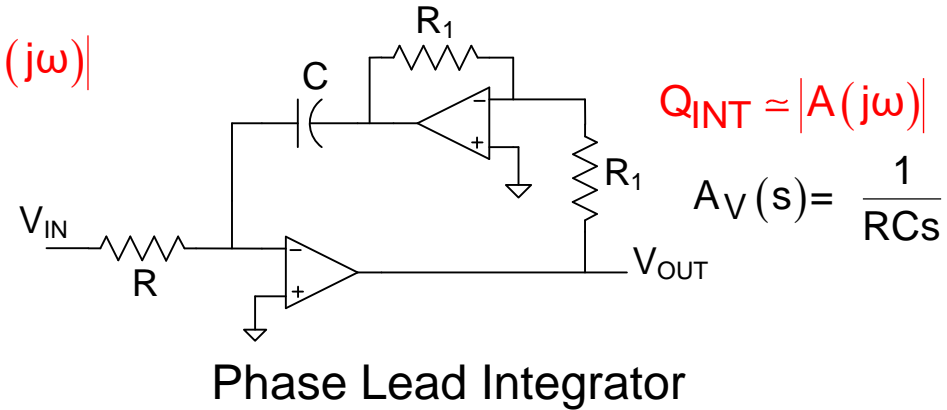
$$A_V(s) = \frac{1}{RCs}$$

Phase Lead Integrator

$$Q_{INT} \simeq \left|A(j\omega)\right|$$

$$A_V(s) = \frac{1}{RCs}$$

Modified High-Q Noninverting

$$Q_{INT} \simeq -\left|A(j\omega)\right|^3$$

$$A_V(s) = \frac{1}{RCs}$$

High-Q NonInverting

$$A_V(s) = \frac{1}{RCs}$$

Zero Sensitivity Noninverting

$$A_V(s) = \frac{1}{RCs}$$

Modified Miller Noninverting

$$Q_{INT} \simeq -\left|A(j\omega)\right|$$

$$A_V(s) = \frac{1}{RCs}$$
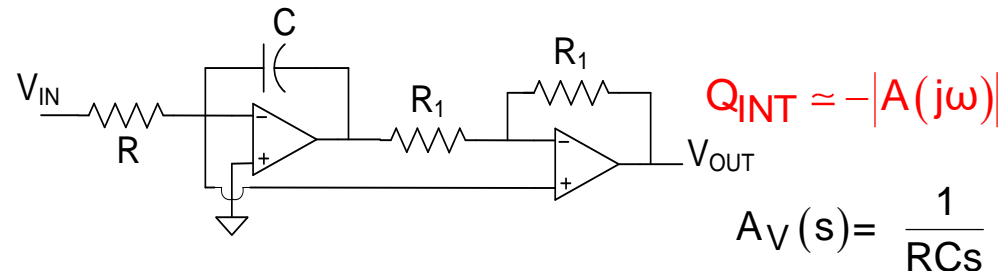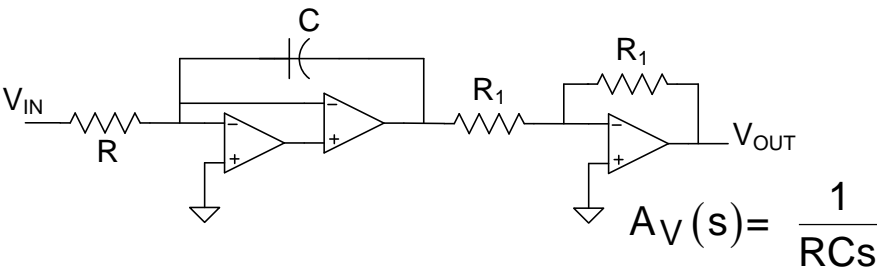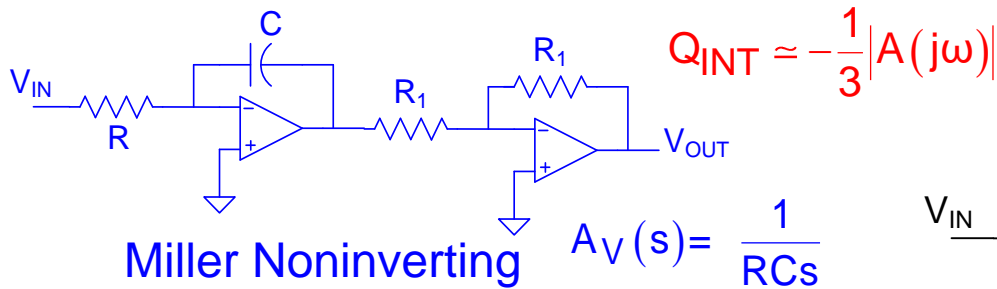
Improving Integrator Performance:

1. Compensate Integrator
2. Use better integrators
3. Use phase-lead and phase/lag pairs

- These methods all provide some improvements in integrator performance

- But both magnitude and phase of an integrator are important so focusing only on integrator Q factor only may only improve performance to a certain level

- In higher-order integrator-based filters, the linearity in $1/\omega$ of the integrator gain is also important. The integrator magnitude and Q factor at ω_0 ignore the frequency nonlinearity that may occur in the $1/\omega$ dependence

- There is little in the literature on improving the performance of integrated integrators within a basic class. At high frequencies where the active device performance degrades, particularly in finer-feature processes, there may be some benefits that can be derived from architectural modifications along the line of those discussed in this lecture

Stay Safe and Stay Healthy !

End of Lecture 27